

Inductive Conductivity Measurement of Seawater

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Abstract: Approximately seventy one percent (70.9%) of the surface of the Earth comprises seawater. Evaporation of seawater controls the moisture content of the atmosphere. The moisture content of the atmosphere controls the clouds, the cloud structure and ultimately the local and global climate through the mechanism of heat transfer by conduction, convection and radiation. All of these mechanisms are ultimately controlled by the properties and heat content of the vast volumes of seawater that underlie the atmosphere. This paper presents a methodology that can be employed to accurately determine the conductivity of seawater. This measurement technique accurately facilitates the estimation of the dissolved mineral content and also the phase transition point of the measured volume of seawater.

Keywords: seawater, conductivity, inductive, dissolved minerals, salinity.

1. Introduction

The three most important properties of seawater are the temperature (T), the salinity (S), and the pressure (P). Of these three properties, the salinity (S) is in many ways the most significant property. The dissolved salts in the water salt solution, mostly sodium chloride, raise the density, raise the conductivity, and lower the freezing point of the resulting liquid (seawater) from that of the original liquid (fresh water).

Since the conductivity of the resulting solution directly correlates with the concentration of dissolved salts, it is important that a method be available that can rapidly and accurately measure the conductivity of seawater. The model presented herein demonstrates a new approach to the methodology of inductive conductivity measurement for liquids. In this case, the liquid of choice is seawater.

2. Inductive Conductivity Measurement Theory

This inductive conductivity measurement technique derives from employing *Faraday's*

Law of Induction, in the configuration of an O-Core transformer. The transformer comprises a multi-turn primary coil, a multi-turn secondary coil, and a ferromagnetic O-Core. The transformer configuration is shown in Figure 1.

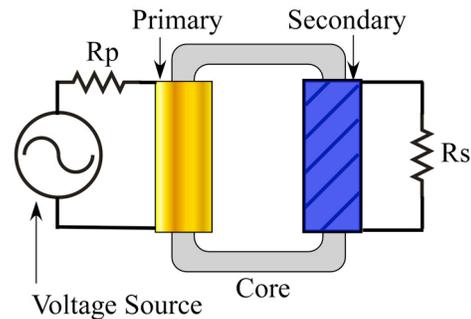


Figure 1. O-Core Transformer Coils, Core and Circuit Configuration.

This model is derived from the ecore transformer model distributed with the ACDC Module of COMSOL Multiphysics. The ecore transformer model and this model are both first principles approximation models, in that they incorporate nonlinearity, through saturation effects in the BH core response curve, but they do not calculate secondary effects, such as intra-coil eddy currents and intra-core hysteresis losses.

This model differs from the ecore transformer model, in that the two multi-turn coils are physically separated from each other and are located on two different portions of the O-Core (o-core), as compared to being overlaid on the central portion of the ecore device.

The primary coil of this model has a copper conductor. The secondary coil of this model has a seawater (liquid) conductor (constrained by thin, non-conductive tubing). The seawater comprising the secondary coil is analyzed as a simple conductor, ignoring heating effects and the associated induced convection effects, as a first approximation.

2.1 Transformer Theory

The primary coil of the o-core transformer is driven by an applied voltage source V_{ac} {5}. The applied voltage source causes a current to flow in the primary coil. The flowing current generates a magnetic flux around the coil that is proportional to the number of turns in the coil.

$$V_{ac} = -N_p \frac{d\phi}{dt} \quad 2.1$$

Where: V_{ac} = Applied Voltage
 N_p = Number of Turns in the Primary
 $d\phi$ = Differential Magnetic Flux
 dt = Differential Time

In integral form, Equation 2.1 is:

$$V_{ac} = -\frac{d}{dt} \int_s \vec{B} \cdot \vec{n} da \quad 2.2$$

Where: V_{ac} = Applied Voltage
 \vec{B} = Flux Density
 \vec{n} = Unit Normal Vector
 da = Differential Area
 dt = Differential Time

The magnetic field intensity \vec{H} gives rise to the magnetic flux density, as follows:

$$\vec{B} = \mu_r \mu_0 \vec{H} \quad 2.3$$

Where: \vec{B} = Magnetic Flux Density
 \vec{H} = Magnetic Field Intensity
 μ_r = Relative Permeability
 μ_0 = Free Space Permeability

2.2 Inductance Theory

Figure 2 shows a magnetic transformer circuit with a primary coil, a secondary coil, a mean core length, and a cross-section area.

Ampere's Law states that the line integral of the magnetic field intensity around a closed contour is equal to the total current enclosed {6}.

$$\oint_C \vec{H} \cdot d\vec{l} = i \quad 2.4$$

In the case where there are a discrete number of loops, as in the case of a closed transformer core multi-turn coil, Equation 2.4 becomes:

$$\sum_N H \cdot l = Ni \quad 2.5$$

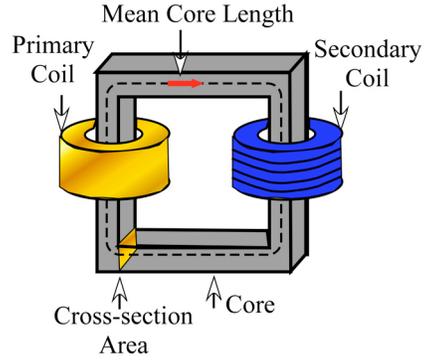


Figure 2. Magnetic Transformer Circuit Showing the Primary Coil, the Secondary Coil, the Mean Core Length, and Cross-section Area.

Assuming that the transformer of Figure 2 has an applied load, as shown in Figure 1, then the voltage applied to the primary coil induces a magnetic flux in the ferromagnetic o-core. The flux in the o-core, in turn, induces a voltage in the secondary coil {7}:

$$V_s = -N_s \frac{d\phi}{dt} \quad 2.6$$

Where: V_s = Induced Voltage
 N_s = Number of Turns in the Secondary Coil
 $d\phi$ = Differential Magnetic Flux
 dt = Differential Time

Since the secondary coil, in this case, has significant internal impedance, the induced voltage will be divided between the internal impedance of the coil and the external impedance of the load resistor.

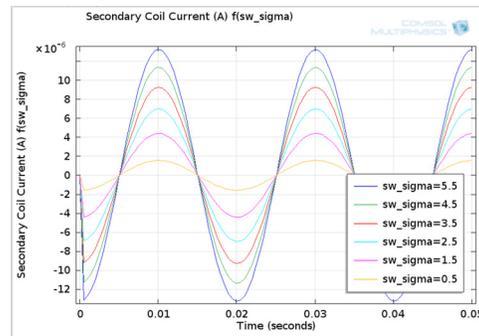


Figure 3. Secondary Coil Output Voltage as a Function of Seawater Conductivity Change.

Thus, as the conductivity of the seawater changes, the voltage developed across the load resistor changes accordingly. Figure 3 shows the calculated values obtained using the physical and component parametric values incorporated into this model.

3. Use of COMSOL Multiphysics

In the model presented in this paper, COMSOL Multiphysics is employed to build a parametrically swept model of an o-core inductive conductivity measurement sensor for seawater. This sensor model is built using the COMSOL Multiphysics Magnetic Fields (mf) and Electric Circuit (cir) physics interfaces.

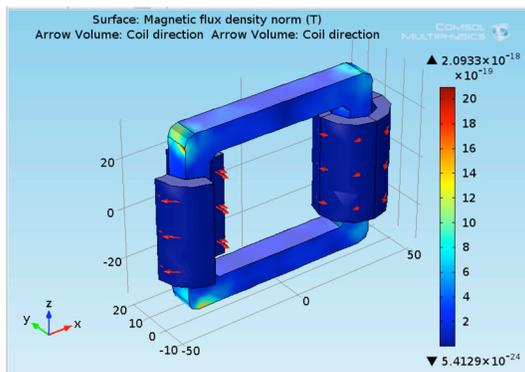


Figure 4. Magnetic flux density in the O-core and the currents in each multi-turn coil.

Figure 4 shows the calculated magnetic flux density solution in the core and the current flow in each of the coils of the dual multi-turn coil o-core inductive conductivity measurement sensor. Multiple turns of copper comprise the primary coil. Multiple turns of seawater comprise the secondary coil.

3.1 Building the Model

Open the COMSOL Multiphysics desktop. In the Model Wizard, Build a 3D model by first selecting **3D**. In the next step, first add the **AC/DC>Magnetic Fields (mf)** interface, then add the **AC/DC>Electric Circuit (cir)** interface. In the next and final initial step, select **Preset Studies for Selected Physics>Time Dependent**. Then **Click** the **Finish** flag.

Figure 5 shows the selection steps necessary for each geometrical item required to build the o-core inductive conductivity sensor geometry.

The dimensions for each item, in mm, are shown in the Appendix in Table 1, Table 2, and Table 3.

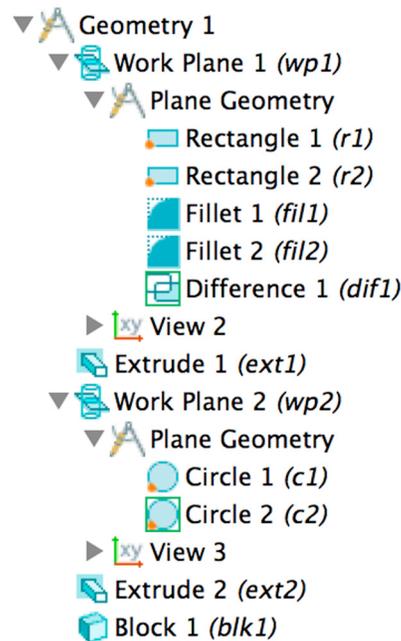


Figure 5. Geometry Build Steps in Model Builder

The o-core is the fundamental element of the sensor and is built by first creating an *xz*-Work Plane at the *y*-coordinate 10 mm. Two rectangles are then built and filleted in the *xz*-Work Plane. The Boolean operation Difference is then used to form the cross-section of the o-core geometry in the *xz*-Work Plane. Next, by using the Extrude operation to add depth in the *y* direction by 10 mm, the solid volume of the o-core is formed.

Once the o-core is formed, it is necessary to add the two multi-turn coils. The two multi-turn coils are built by first creating an *xy*-Work Plane at the *z*-coordinate -20 mm. Two circles are then built in the *xy*-Work Plane. The dimensions for each item, in mm, are shown in the Appendix in Table 2. The first circle, *c1*, surrounds the left-most vertical-section of the o-core. The second circle, *c2*, surrounds the right-most vertical-section of the o-core. The circles are configured with a 5 mm layer thickness. Once the circles have been constructed, the circles are then converted to 40 mm long coils through the use of the Extrude command.

A surrounding Block is then added to complete the final step in the creation of the model geometry. Table 3, in the Appendix, shows the dimensions, location and orientation

of the surrounding block blk1. Figure 6 shows the configuration of the final geometry.

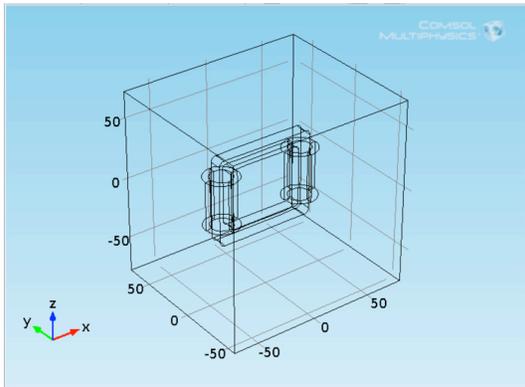


Figure 6. O-Core Inductive Conductivity Sensor

3.2 Global Parameters

Right-click Global Definitions and Select Parameters. Enter the Global parameters that are shown in Table 4 in the Appendix.

3.3 Explicit Selections

In order to easily configure the properties of subsections of the model, an explicit should be selected and named for the desired domain or boundary being configured, as shown in Figure 7. The Core explicit is for the core domain. The Primary Winding explicit is for the primary winding domains. The Secondary Winding explicit is for the secondary winding domains. The Windings explicit is for all winding domains. The Primary Insulation explicit is for all the insulated primary boundaries. The Secondary Insulation explicit is for all the insulated secondary boundaries.

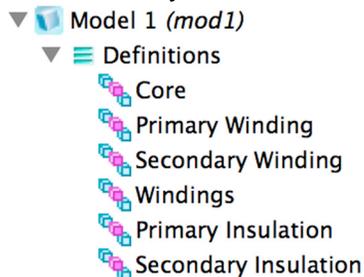


Figure 7. Selected Geometrical Explicit 1 - 6

3.4 ACDC Magnetic Fields User Interface

The first necessary step, when using the ACDC Magnetic Fields User Interface, is to determine the Discretization of the Magnetic Vector Potential. Click on the Show button in the Model Builder Toolbar and be sure that Discretization is checked.

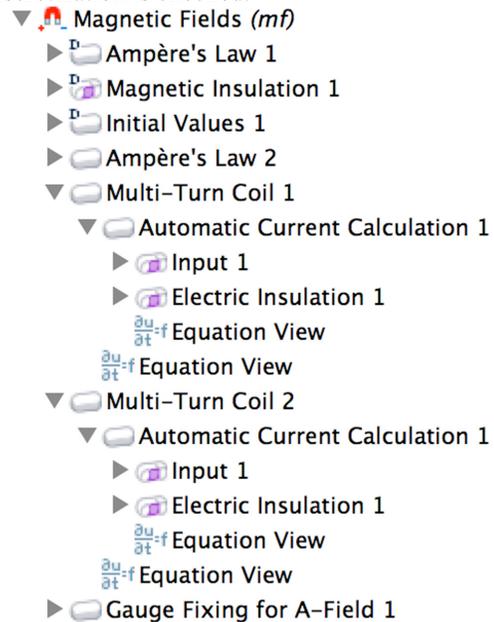


Figure 8. Magnetic Fields User Interface

Click on Magnetic Fields (mf). In the Magnetic Fields window, under the Discretization tab, select Linear {8} from the Magnetic vector potential pull-down menu. This action changes the magnetic vector potential from Quadratic (default) to Linear.

Figure 8 shows the following steps used to properly configure this model.

Right-Click Magnetic Fields (mf) > Select Ampere's Law. In the Ampere's Law settings window, use the Selection list and Select Core.

Right-Click Magnetic Fields (mf) > Select Multi-Turn Coil. In the Multi-Turn Coil settings window, use the Selection list and Select Primary Winding. In the same window, locate the Coil Type and Select Numeric from the pull-down menu. Insert N_p in the number of turns edit field. Select Circuit (current) from the Coil excitation pull-down menu.

Right-Click Magnetic Fields (mf) > Multi-Turn Coil 1 and Select Automatic Current Calculation.

Right-Click Magnetic Fields (mf) > Multi-Turn Coil 1 > Automatic Current Calculation 1 and Select Input.

In the Input settings window, Clear all Boundary Selections and Enter Boundary 22. The location of Boundary 22 is shown in Figure 9.

Next, Right-Click Magnetic Fields (mf) > Select Multi-Turn Coil. In the Multi-Turn Coil settings window, use the Selection list and Select Secondary Winding. In the same window, locate the Coil Type and Select Numeric from the pull-down menu. Insert Ns in the number of turns edit field. Enter sw_sigma as the value of the conductivity of the Multi-Turn Coil 2. Enter $4.6e-5[m^2]$ as the area of the coil wire cross-section. Select Circuit (current) from the Coil excitation pull-down menu.

Right-Click Magnetic Fields (mf) > Multi-Turn Coil 2 and Select Automatic Current Calculation.

Right-Click Magnetic Fields (mf) > Multi-Turn Coil 2 > Automatic Current Calculation 1 and Select Input.

In the Input settings window, Clear all Boundary Selections and Enter Boundary 50. The location of Boundary 50 is shown in Figure 10.

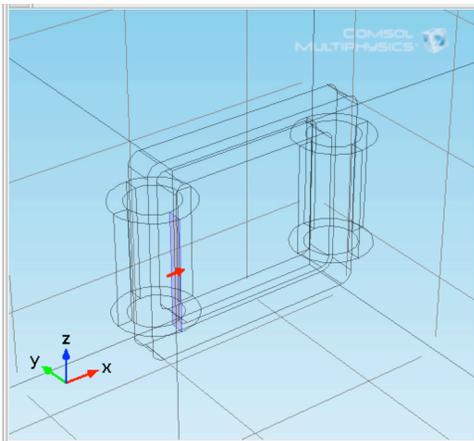


Figure 9. Primary Winding Input Terminal

Finally, Right-Click Magnetic Fields (mf) > Select Gauge Fixing for A-Field. In the Gauge Fixing Settings window, Select > All domains from the pull-down menu. This choice is necessary and avoids singularities {9}.

3.5 ACDC Electrical Circuit User Interface

In the current section of this paper, the external circuitry is added to the inductive conductivity measurement sensor, as shown in Table 5.

Right-Click Electric Circuit (cir) > Select and configure each Desired Circuit Element, as listed in Table 5 (in sequence), as shown in Figure 11.

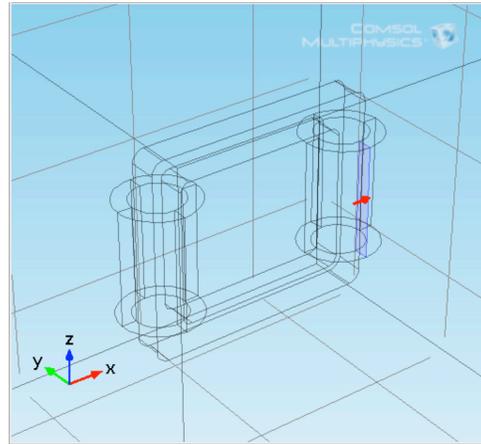


Figure 10. Secondary Winding Input Terminal

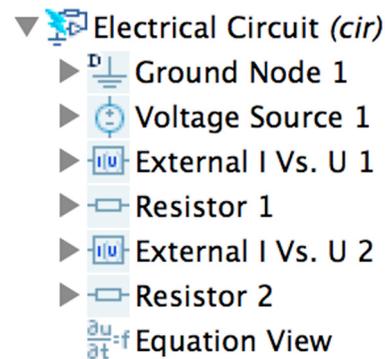


Figure 11. Electric Circuit Elements

The first added element is Voltage Source 1. On the Voltage Source settings page enter the Node names (connection points), $V_{src} = V_{ac}$ and $f = \omega$.

The second added element is External I Vs. U 1. On the External I Vs. U settings page enter the Node names (connection points), and Select **V=Coil voltage (mf/mtcd1)** from the Electric potential pull-down menu.

The third added element is Resistor 1. On the Resistor settings page enter the Node names (connection points), and Enter R_p in the Resistance edit window.

The fourth added element is External I Vs. U 2. On the External I Vs. U settings page enter the Node names (connection points), and Select **V= Coil voltage (mf/mtd2)** from the Electric potential pull-down menu.

The fifth added element is Resistor 2. On the Resistor settings page enter the Node names (connection points), and Enter Rs in the Resistance edit window.

3.6 Materials

The four materials used in this model are detailed in Table 6 and shown in Figure 12. The conductivity of Air is modified to 0.01[S/m] for stability in convergence. The conductivity of the Soft Iron Core is modified to 5.5[S/m] for stability in convergence, also.

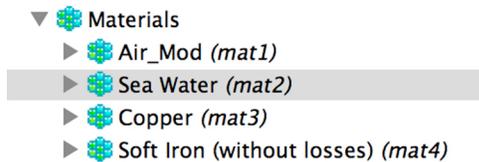


Figure 12. Materials

4.0 Mesh

Click>Mesh 1. In the Mesh Settings window Select>Extra coarse from the Element size list. Right-click>Mesh 1 Select Free Tetrahedral. In the Free Tetrahedral settings window Select Entire Geometry. Right-click>Free Tetrahedral Select Size. In the Size 1 settings window, Select>Domain and Paste 2-10 in the Selection window. Click the Custom button. Enter 8 in the Maximum element size edit window. Click>Build All. The Mesh is shown in Figure 13.

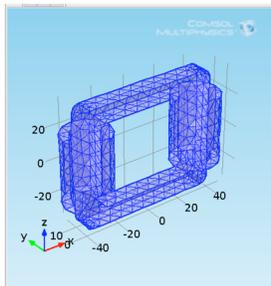


Figure 13. Meshed Model

5.0 Study

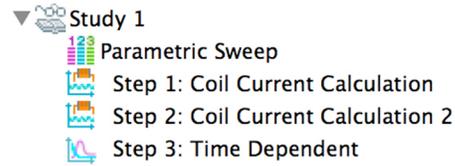


Figure 14. Study 1 Sequence

Figure 14 shows the Study 1 sequence followed to solve this model. This sequence differs from those used for the ecore transformer by the addition of the Parametric Sweep to the calculation sequence.

6.0 Results

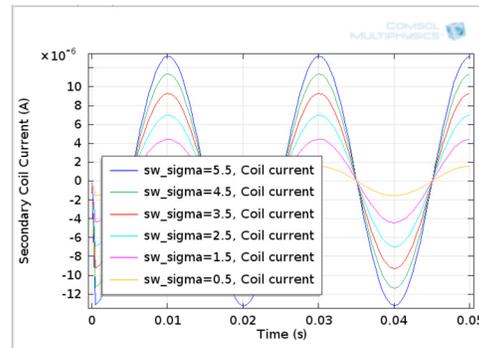


Figure 15. Secondary Voltage Vs. Conductivity

Figure 15 shows that changes in the conductivity of seawater are easily detectable using this method.

7. Conclusions

This model demonstrates that changes in seawater conductivity are readily measured with an inductive methodology. For a detailed overview of the many applications that utilize the conductivity of seawater and other liquids, I refer the reader to the literature.

8. References

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9. Appendix

Table 1: Work Plane 1 Parameters in mm

Item	Type	Location		
Work Plane	xz-plane	y=10		
Item	Type	Width	Height	Base
R1	Solid	80	70	Center
R2	Solid	60	50	Center
Item	Radius			
Fillet R1	4			
Fillet R2	3			
Item	Distance			
Extrude	10			

Table 2: Work Plane 2 Parameters in mm

Item	Type	Location		
Work Plane	xy-plane	-20		
Item	Type	Radius	Base	Layers
C1	Curve	14	Center	Layer 1
			xw=-35	5

Item	Type	Radius	Base	Layers
			yw=5	
C2	Curve	14	xw=35	Layer 1
			yw=5	5
Item	Distance			
Extrude	40			

Table 3: Environmental Block Parameters in mm

Item	Type	Width	Depth	Height
blk1	Solid	150	130	150
Position	Center			
Axis	z-axis			

Table 4: Global Parameters

Name	Expression	Value
Rp	100[ohm]	Primary R
Rs	4.8552e2[ohm]	Secondary R
Np	100	Primary Turns
Ns	5	Secondary Turns
nu	50[Hz]	Frequency
Vac	25[V]	Applied Voltage
sw_sigma	5.5[S/m]	Saltwater Conductivity

Table 5: Electrical Circuit Elements

Element	Value V	Value f	Node p	Node n
Voltage Source	Vac	nu	1	0
External I Vs. U 1	Coil Voltage (mf/mtd1)		2	1
Resistor 1	Rp		0	2
External I Vs. U 2	Coil Voltage (mf/mtd2)		3	0
Resistor 2	Rs		0	3

Table 6: Materials

Material	Domain Assignment
Air	1(Environmental)
Seawater	7-10 (Secondary)
Copper	2,3,5,6 (Primary)
Soft Iron	4 (Core)